11/7/19

M159 (Continued)

SUDD:

2 decrement example

$$q_x^{(2)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(2)} \cdot dt$$

$$= \int_0^1 \underbrace{{}_tp_x^{\prime(1)}}_{=1-t\cdot q_x^{\prime(1)}} \cdot \underbrace{{}_tp_x^{\prime(2)} \cdot \mu_{x+t}^{(2)}}_{\overset{UDD}{=} \text{ constant } = q_x^{\prime(2)}} dt$$

$$= q_x^{\prime(2)} \cdot \int_0^1 (1 - t \cdot q_x^{\prime(1)}) dt$$

$$q_x^{(2)} = q_x^{\prime(2)} \cdot \left(1 - \frac{q_x^{\prime(1)}}{2}\right)$$

Likewise $q_x^{(1)} = q_x^{\prime(1)} \cdot \left(1 - \frac{q_x^{\prime(2)}}{2}\right)$

H.W. Exercises:

#13) $p_x^{(\tau)} = 0.72 \Longrightarrow q_x^{(\tau)} = 0.28$

(a) MUDD

$${}_tp_x^{\prime(j)} = \left[{}_tp_x^{(\tau)}\right]^{\left(q_x^{(j)}/q_x^{(\tau)}\right)}$$

Use $t=1$

$j=1$

$$\underset{\underset{0.9 \text{ (given)}}{\parallel}}{p_x^{\prime(1)}} = \left[p_x^{(\tau)}\right]^{\left(q_x^{(1)}/q_x^{(\tau)}\right)}$$

$$0.9 = (0.72)^{\left(q_x^{(1)}/0.28\right)}$$

$$\Longrightarrow q_x^{(1)} = 0.28 \cdot \frac{\ln(.9)}{\ln(.72)} = 0.0898\cdots$$

$q_x^{(2)}$: (Use totals)

$$q_x^{(\tau)} = 0.28 = 0.0898\cdots + q_x^{(2)}$$

$$\Longrightarrow q_x^{(2)} = 0.1901\cdots$$

(b) SUDD

$$q_x^{(1)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(1)}\, dt$$

$$= \int_0^1 \underbrace{{}_tp_x^{\prime(2)}}_{\substack{=1-t\cdot q_x^{\prime(2)} \\ =1-0.2t}} \cdot \underbrace{{}_tp_x^{\prime(1)} \cdot \mu_{x+t}^{(1)}}_{= \text{constant} = q_x^{\prime(1)} = \cancel{0.2}\ 0.1 \text{ (given)}}\, dt$$

$q_x^{\prime(2)} = .02$ (given)

$$= 0.1 \cdot \int_0^1 (1-0.2t)\, dt$$

$$= 0.1 \cdot (1-0.1) = 0.09$$

$$q_x^{(2)} = q_x^{(\tau)} - q_x^{(1)} = 0.28 - 0.09 = 0.19$$

UDD Facts:

$${}_tp_x \cdot \mu_{x+t} = q_x$$

$${}_tq_x = t \cdot q_x \Longrightarrow {}_tp_x = 1 - t \cdot q_x$$

Discrete Decrement Examples

17) (1) is BOY $\quad q_x^{(1)} = 0.1$

(2) is SUDD $\quad q_x^{(2)} = 0.2$

Since (1) is BOY, then $q_x^{(1)} = q_x^{\prime(1)} = 0.1$

Then $q_x^{(\tau)} = 0.3 \Longrightarrow p_x^{(\tau)} = 0.7 = \underbrace{p_x^{\prime(1)}}_{0.9} \cdot p_x^{\prime(2)}$

$\Longrightarrow p_x^{\prime(2)} = \frac{7}{9}$

$\therefore q_x^{\prime(2)} = \frac{2}{9}$

Same as 17 except (1) is EOY

Since (1) is EOY, then $q_x^{(2)} = q_x^{\prime(2)} = 0.2$

Then $q_x^{(\tau)} = 0.3 \Longrightarrow p_x^{(\tau)} = 0.7 = p_x^{\prime(1)} \cdot (0.8)$

$\Longrightarrow p_x^{\prime(1)} = \frac{7}{8} \Longrightarrow q_x^{\prime(1)} = \frac{1}{8}$

<u>Note</u>: $q_x^{(1)} = p_x^{\prime(2)} \cdot q_x^{\prime(1)}$